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# Item Response Models for Continuous Bounded Responses

with applications in emotion, personality and behavior  
change research



# Foreword

This document summarizes a long evolution of about 20 years of research on item response modeling for emotion, personality and behavior change data. Although I already was quite inclined towards math, I did not come to psychology for the love of psychometrics, which I knew nothing about before studying psychology at University of Rennes. I actually wanted to be a clinical psychologist, and eventually was, working with smokers at the Associative House for Health (1993-1996) and with heroin addicts at the Psychiatric Hospital (1997), in Rennes.

I was not so attracted by the (then widespread) psychoanalytic theory, at least not as far as intervention planning is concerned, and was looking for a conceptual framework that would provide clear and operational criteria for guiding action as a psychologist. The discovery of the Transtheoretical Model of Change was a shock. It seemed to provide everything I needed: Clear criteria for stages and processes of change, and a number of measurement instruments.

In an initial research<sup>1</sup> on a clinical sample of smokers coming for assistance in quitting smoking, I had collected a number of measures (59 tools were used, among which the Transtheoretical questionnaires on stages, processes, decisional balance and self-efficacy, but also Five-Factor personality measures, emotion management scales, field dependence, self-consciousness, to name but a few). My first surprise while examining the data with principal component analysis (PCA) was the omnipresence of circular patterns of points in the observation space, not only with affective data, for which it had been described for long, but also with processes of change data, about which it had not been reported. Even in the affective literature where so-called *circumplexes* were discussed at length, authors were concentrating on variable loading plots and not on the observation euclidian space. Although circular patterns are expected in variable loading plots, where correlations are nothing but angular measures (cosines), I could not get any mathematical meaning out of that strange phenomenon in the observation plot, as extracting orthogonal factors logically means that subjects may virtually have high scores on several factors simultaneously, and so points should be dispersed everywhere. Nor could I give it any general psychological meaning as I could see it happen in so different, cognitive and affective domains.

A paper by van Schuur and Kiers (1994) was illuminating to this respect. They showed that when the response mechanism may be modeled by a nonlinear, inverted-U shaped function, for example when individuals are characterized by ideal positions along a latent dimension, then this pattern occurs in PCA. The paper was descriptive, and neither provided a general mathematical formulation of the phenomenon, nor indicated if it could happen with other mechanisms, but it convinced me that no valid measurement could be constructed in

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<sup>1</sup> Noël, Y. (1996). Evaluation d'une technique d'hypnothérapie du tabagisme : application de l'analyse du dépliage [Assessment of a hypnosis-based technique for smoking cessation: An application of unfolding analysis]. Unpublished PhD. thesis, University of Paris 10, Nanterre, under the supervision of Prof. J.M. Petot.

psychology if not model-based. Without an explicit hypothesis on the response function, we are always at risk to interpret spurious and artifactual factors. Cases are numerous in the literature and this document will provide examples in the field of emotion, personality and behavior change.

Twenty years later, I find myself asking almost the same questions, on the very same topics: What is the structure of human emotion and personality? Are there links between them? How is this related to how we change, for quitting smoking, going on a diet, or resuming exercise? Hopefully the technical responses proposed in this document are a little more elaborated than they were at that time.

One of the simplest and seemingly trivial change in assessment practice proposed in this research is to replace Likert scales by continuous bounded formats (or visual analogue scales) in psychological measurement. Not only do they make possible to develop more parsimonious models, but also make appear some subtle phenomena that could not be made visible with Likert scales. Some of them (a nonlinear relationship between valence and arousal in emotion, bimodality in questionnaire responses to contradictory attitudes, and bifurcation in behavior change) will be described in the following chapters. The factor tradition in psychological measurement has been very successful in analyzing aptitudes and intelligence, but also conveys a form of positive structuralism that is probably not so well suited to the analysis of affect, personality and change, where dynamics of opposing forces is the rule rather than the exception. There is a need in developing more dynamical models of conflicting processes in psychology. It is our view that using continuous bounded responses in an item response model perspective is a step forward in that direction.

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